RTED

A Robust Algorithm for the Tree Edit Distance

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VLDB 2012, Istanbul, Turkey

Free University of Bozen-Bolzano

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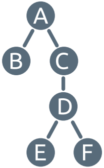
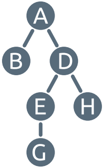
Tree edit distance





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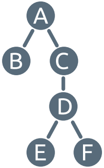

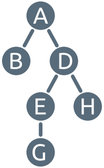

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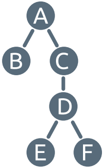

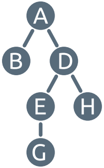

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Three edit operations:

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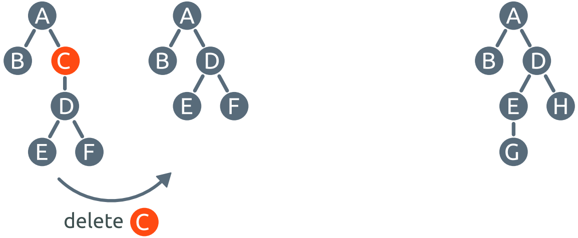

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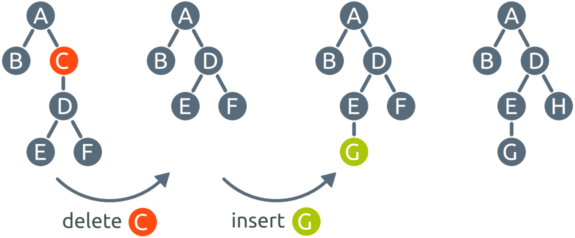

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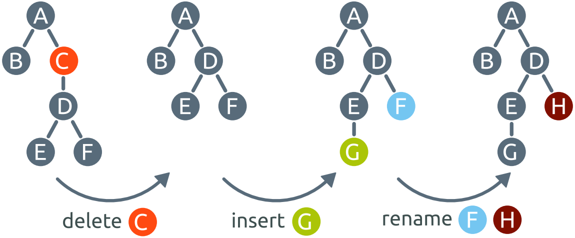

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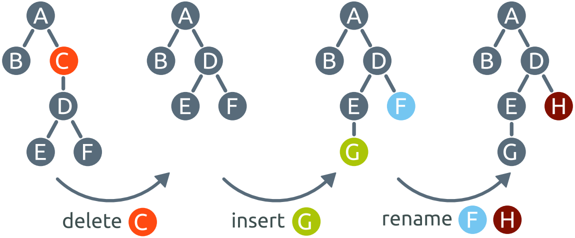

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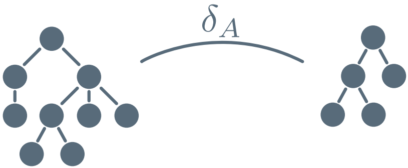
Each operation has a cost.

Cost of distance computation



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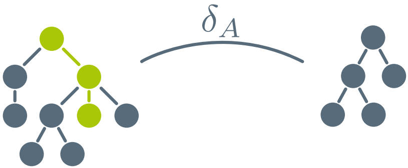

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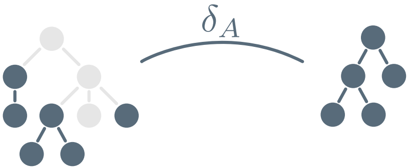

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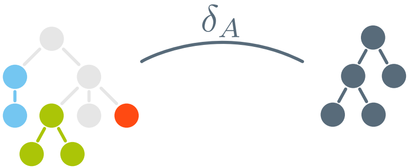

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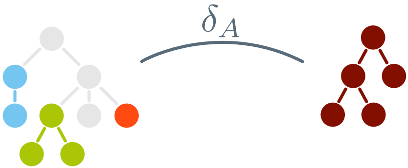

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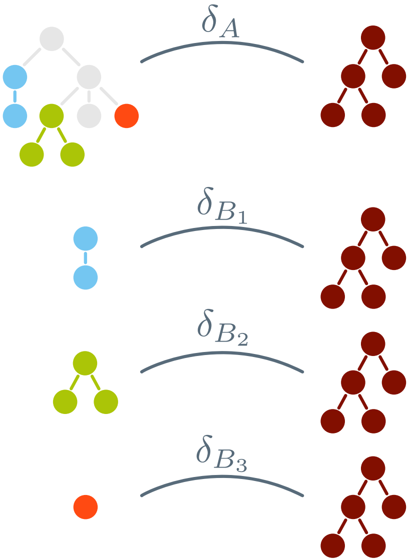

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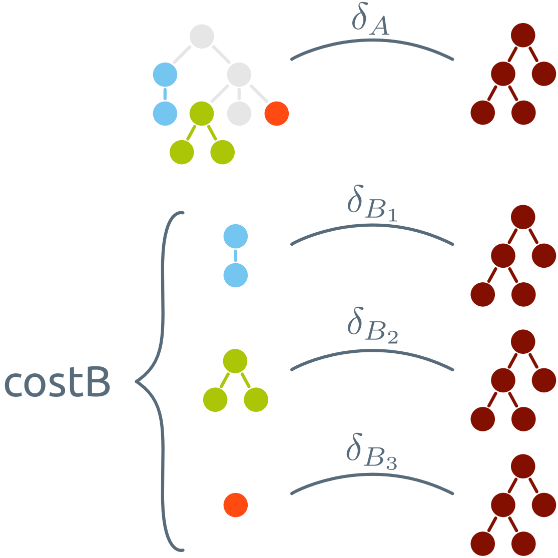

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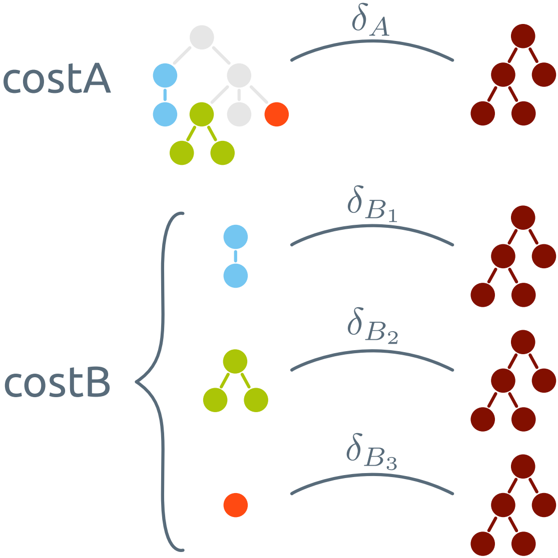

Path strategies

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LRH strategy: allow only left, right and heavy paths.

LRH strategies cover all the strategies used by the state-of-the-art algorithms.

State of the art

Fastest algorithms use LRH strategies.

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- always left path
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State-of-the-art is not satisfactory.

Problem definition

Find a robust algorithm that combines the strengths of the state-of-the-art algorithms excluding their weaknesses.

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- space-efficient: $O(n^2)$ best known space complexity
- optimal runtime: $O(n^3)$ optimal runtime among all possible strategies
- robust: always uses the best possible strategy for a given pair of trees

RTED - robust algorithm for the tree edit distance

RTED = optimal strategy + GTED

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step1: compute optimal LRH strategy for given pair of trees

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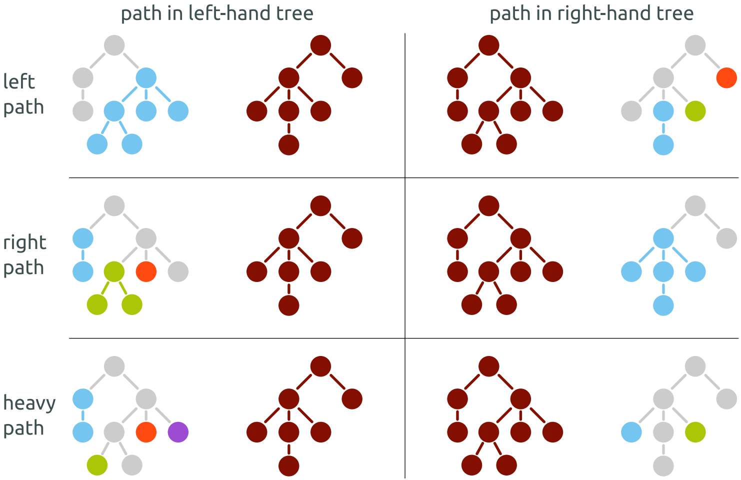
key problem - compute the optimal strategy efficiently

Optimal strategy - six choices



Optimal strategy - cost formula

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Top-down algorithm with memoization.

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We need faster solution.

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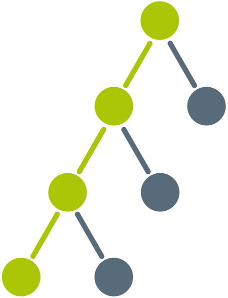

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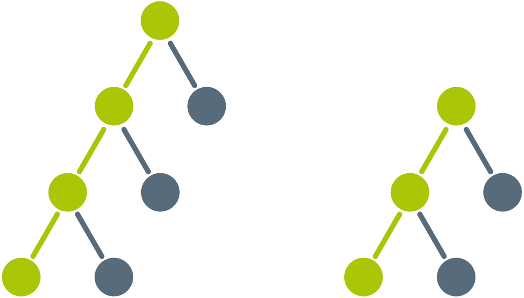

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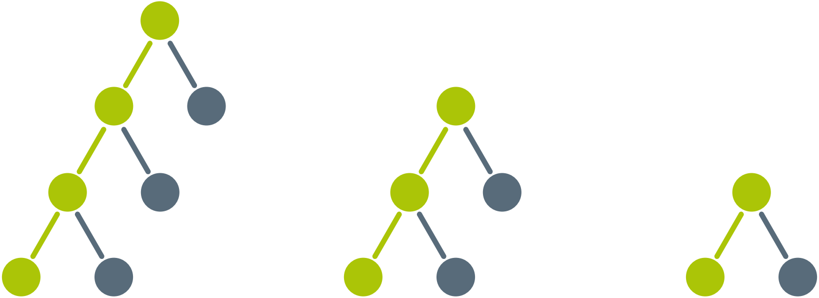

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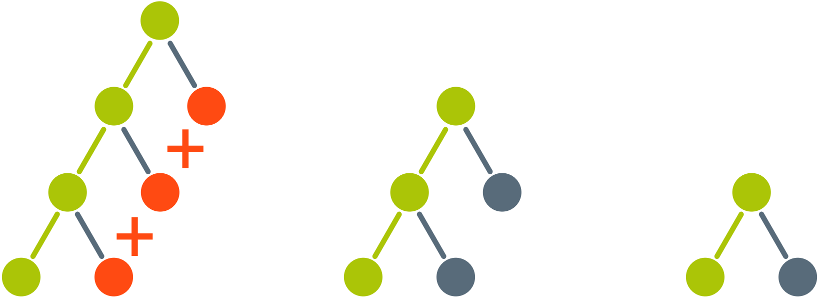

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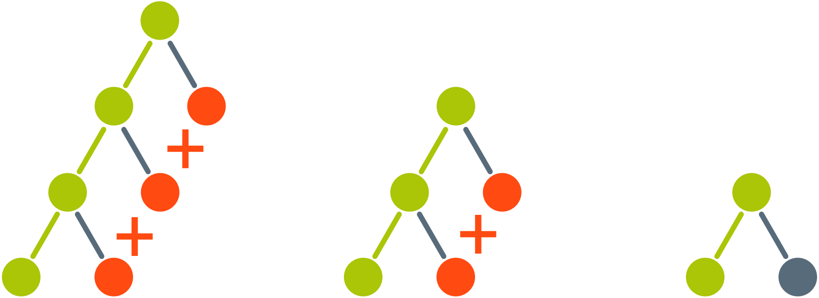

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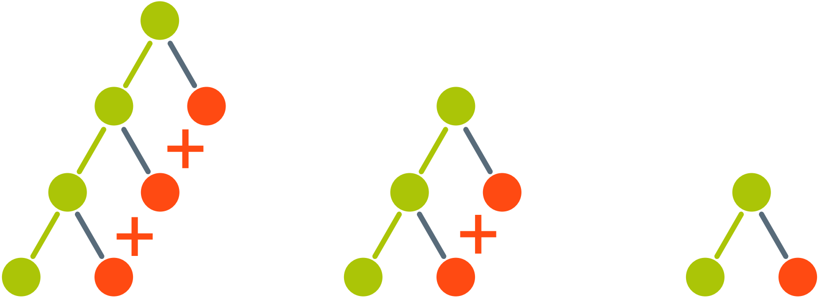

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- costA - constant time
- costB - incrementally computed in constant time - we add only one summand

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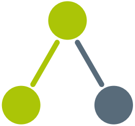

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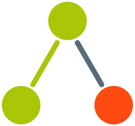

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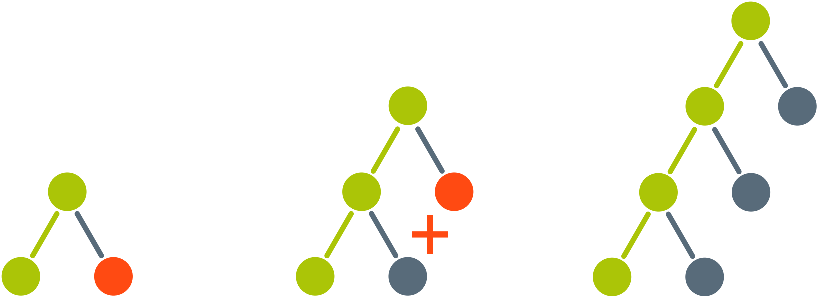

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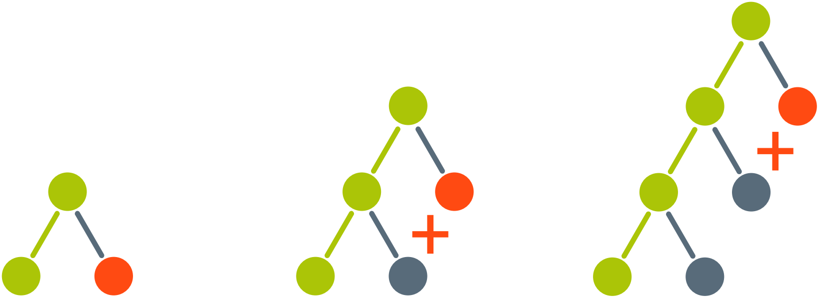

Experiments setup

Algorithms:

- Zhang-L only left paths in one tree
- Zhang-R only right paths in one tree
- Klein-H only heavy paths in one tree
- Demaine-H only heavy paths in bigger tree
- RTED optimal strategy

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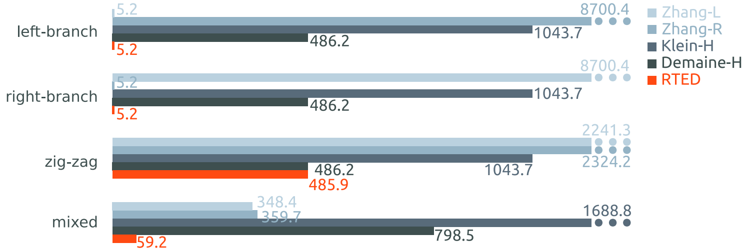
Tree shapes:

left-branch (Zhang-L) | right-branch (Zhang-R) | zig-zag (Demaine-H) | mixed

Scalability of distance computation

synthetic data:

#subproblems$\times 10^6$ for trees of size $\sim$1000 nodes



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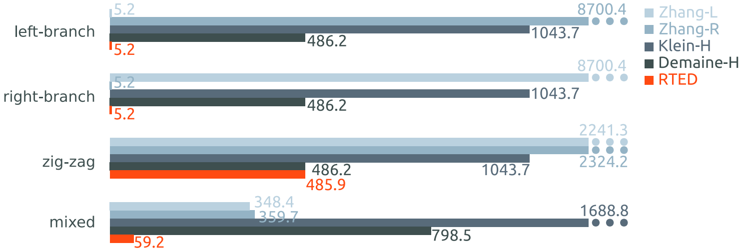
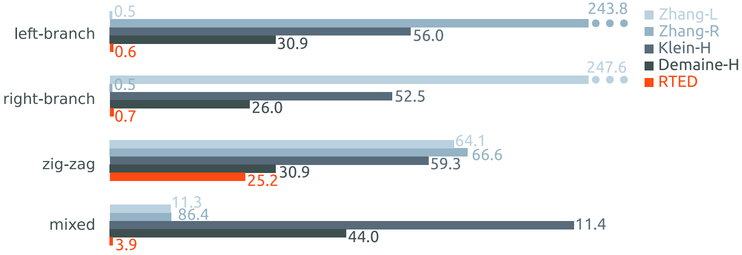


TreeFam (avg depth 14, max depth 158, avg size 95):
#subproblems to the best and worst competitor

- 86.9% of the best competitor
- 5.6% of the worst competitor

Scalability of distance computation

synthetic data:
runtime in seconds for trees of size ∼1000 nodes



Self similarity join

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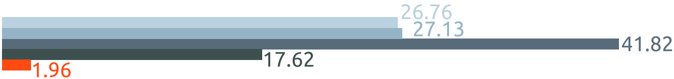

RTED 9-21 times less subproblems

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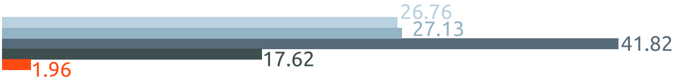

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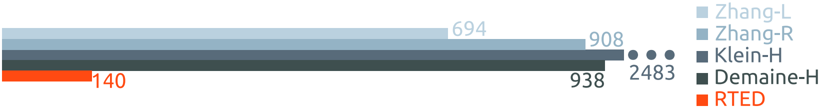
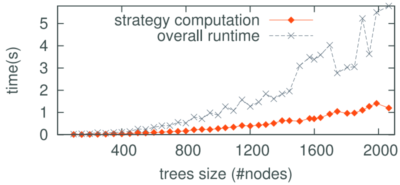


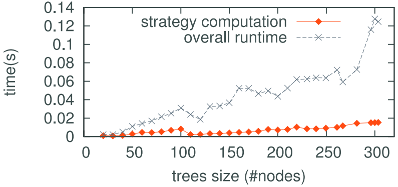
RTED 5-17 times faster

Overhead of strategy computation



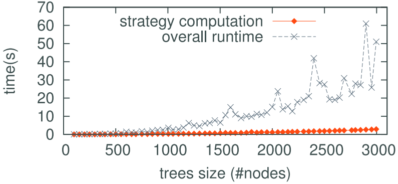
SwissProt

(max depth 4, max fanout 346, avg size 187)



TreeBank

(avg depth 10.4, max depth 35, avg size 68)



synthetic random trees

(varying size, fanout, depth)

Conclusion

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RTED

- uses best possible strategy
- insensitive to tree shape
- theoretical results and experiments prove its efficiency

Conclusion

First experimental evaluation of tree edit distance algorithms.

State-of-the-art algorithms for tree edit distance

- tree shape matters
- choosing algorithms is hard
- wrong choice = large runtime difference

RTED

- uses best possible strategy
- insensitive to tree shape
- theoretical results and experiments prove its efficiency

Open Source release

- http://www.inf.unibz.it/dis/projects/tree-distance-repository/

Future work

Find the mapping of the nodes.

- Tree edit distance is a number.
- In some scenarios the edit sequence is of interest.